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Thermal Stress Analysis of Rectangular Plate due to Convection using Finite Element Method

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Abstract -- This study investigated the thermal stress analysis of a rectangular plate due to heat transfer by conduction and convection. It is subjected that convection takes place from the extreme edge $(x = a)$, whereas the initial edge $(x = 0)$ is thermally insulated and the initial and extreme edges $(y = 0,\ y = b)$ are at constant temperature T_0. The initial temperature of the rectangular plate is kept at T_i. The governing heat conduction equation has been solved by using finite element method. The finite element formulation for thermoelastic stress analysis has been obtained on the basis of classical theory of thermo elasticity. The results of temperature change, displacement and thermal stress have been computed numerically and illustrated graphically.

***Index terms*--Boundary value problem, Heat transfer analysis, Finite element method, Finite difference method, Thermal stresses Analysis.**

I. INTRODUCTION

Heat transfer is a phenomenon which occurs due to the existence of the temperature difference within a system or between two different systems, in physical contact with each other. The heat generated may be dissipated to another body or to the surrounding through conduction, convection and radiation which are collectively termed as 'modes of heat transfer'. Heat transfer by convection is given by Newton's law of cooling which states that, "the rate of heat transfer by convection between a surface and a surrounding is directly proportional to the surface area of heat transfer and also to the temperature difference between them". It can be mathematically be expressed as, $Q_{convection} = h\ A_s\ (T_s - T_a)$ where A_s is the heat transfer surface area, h is the convection heat transfer coefficient, T_s is the surface temperature and T_a is the surrounding temperature [6]. When temperatures T_s and T_a are fixed by design considerations, it is obvious that there are only two ways by which the rate of heat transfer can be increased, i.e., one by increasing the heat transfer coefficient h and the other by increasing the surface area A_s. David et al. [2] calculate convective heat transfer, predict convection cooling and flow in electric machines. Flow network analysis, is used to study the ventilation inside the machine is presented. This paper provides guidelines for choosing suitable thermal and flow network formulations and setting any calibration parameters used. The finite element method has become a powerful tool for the numerical solution of a wide range of engineering problems. The finite element method (FEM) is a computational technique used to obtain approximate solution of the boundary value problems (BVP) in engineering. BVP or field problem is a mathematical problem in which one or more dependent variables must satisfy a differential equation everywhere within a known domain of independent variables and satisfy the specific boundary conditions. The basic idea of finite element method is develop by Turner et al. [1], to find the approximate solution of a complicated problem by replacing into a simpler problem rather than the exact solution. In the finite element method it will often be possible to improve or refine the approximate solution by spending more computational effort. Dechaumphai et al. [2] used Finite Element Analysis procedure for predicting temperature and thermal stresses of heated products. This paper deals with the realistic problem of the thermal stress analysis of isotropic rectangular plate subjected to the convective heat transfer from the edge $(x = a)$, where as the initial edge $(x = 0)$ are thermally insulated and the remaining edges $(y = 0,\ y = b)$ are at constant temperature T_0. The initial temperature of the plate is kept at T_i. The governing heat conduction equation has been solved by using finite element method. The temperature distribution, displacement analysis, thermal stress analysis and the corresponding finite element formulation are described. The Matlab programming is used to evaluate the temperature change, displacement at different nodes and thermal stresses along *x*, *y* and resultant direction in the rectangular plate. The results presented in this manuscript have better accuracy since numerical calculations have been performed for discretization of the large number of elements in rectangular plate.

II. MATHEMATICAL MODEL

Fig. 1 shows the schematic sketch of rectangular plate with given boundary condition. The governing differential equation of two dimensional heat conduction equations in a thin rectangular plate occupying the space $\{D: 0 \le x \le a, 0 \le y \le b\}$ is given as

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = c\rho\frac{\partial T}{\partial t} \qquad (1)$$

where $\rho,\ c,\ k$ is the density of the solid, specific heat and thermal conductivity of material used for the rectangular plate.

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The boundary conditions on the rectangular plate are as follows:

$$\frac{\partial T}{\partial x} = 0, \text{ at } x = 0, \ 0 \le y \le b \quad (2)$$

$$k\frac{\partial T}{\partial x} = -h(T_s - T_a), \text{ at } x = a, \ 0 \le y \le b \quad (3)$$

$$T = T_0, \text{ at } y = 0, \ y = b, \ 0 \le x \le a \quad (4)$$

$$T(x, y, t) = T_i, \text{ at } t = 0, 0 \le x \le a, 0 \le y \le b \quad (5)$$

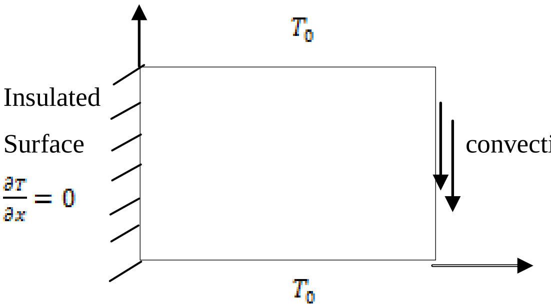

Fig. 1: "Geometry showing 2D rectangular plate subjected to convection"

III. SOLUTION OF THE PROBLEM

A. Galerkin's Finite Element Approach

Applying Galerkin's finite element approach to reduces the given Partial Differential Equation in to a system of algebraic equations. Using the approach by the David Hutton [9], the value of temperature distribution *T(x, y, t)* in the element is described as

$$T(x, y, t) = \sum_{i=1}^{M} N_i(x, y) T_i(t) = [N]^T\{T\} \quad (6)$$

where, $N_i(x, y)$ is the interpolation or shape function associated with nodal temperature T_i, $[N]$ is the row matrix of interpolation functions, $\{T\}$ is the column matrix (vector) of nodal temperatures. Now the two dimensional isotropic rectangular plate residual equations corresponding to equation (1) for all $i = 1, 2, \dots \dots M$ is given by

$$\iint_A N_i(x, y)\left[\left(k\frac{\partial^2 T}{\partial x^2} + k\frac{\partial^2 T}{\partial y^2}\right) - \rho c\frac{\partial T}{\partial t}\right] dA = 0$$

Integrating by part the first two terms of the above equation

$$\iint_A \left[k\frac{\partial N_i(x,y)}{\partial x}\frac{\partial T}{\partial x} + k\frac{\partial N_i(x,y)}{\partial y}\frac{\partial T}{\partial y} + \rho c N_i(x, y)\frac{\partial T}{\partial t}\right] dxdy = \int_S \left[kN_i(x, y)\frac{\partial T}{\partial y}\right]_{y=0}^{b} dx + \int_S \left[kN_i(x, y)\frac{\partial T}{\partial x}\right]_{x=0}^{a} dy$$

Applying the boundary conditions of equation (2) and equation (3)

$$k\iint_A \frac{\partial N_i(x,y)}{\partial x}\frac{\partial T}{\partial x} dA + k\iint_A \frac{\partial N_i(x,y)}{\partial y}\frac{\partial T}{\partial y} dA + \rho c\iint_A N_i(x, y)\frac{\partial T}{\partial t} dA = k\int_S \left[N_i(x, y)\frac{\partial T}{\partial y}\right]_{y=b} dx - k\int_S \left[N_i(x, y)\frac{\partial T}{\partial y}\right]_{y=0} dx - h\int_S [N_i(x, y)(T_s - T_a)]_{x=a} dy$$

Using equation (6), the two dimensional above equation reduces to

$$k\iint_A \frac{\partial N_i(x,y)}{\partial x}\frac{\partial N_i(x,y)}{\partial x}^T\{T\}dxdy + k\iint_A \frac{\partial N_i(x,y)}{\partial y}\frac{\partial N_i(x,y)}{\partial y}^T\{T\}dxdy + \rho c\iint_A N_i(x,y)[N_i(x,y)]^T\{\dot{T}\}dxdy = k\int_{x=0}^{a} N_i(x, b)\left[\frac{\partial N_i(x,b)}{\partial y}\right]_{y=b}\{T\}dx - k\int_{x=0}^{a} N_i(x, 0)\left[\frac{\partial N_i(x,0)}{\partial y}\right]_{y=0}\{T\}dx + \int_S N_i(a, y)h(T_a - N_i(a, y)\{T\})dy$$

This is of the form

$$([K_1] - [K_2] + [K_3] + [K_4])\{T\} + [C]\{\dot{T}\} = \{f_g\}$$

$$[K]\{T\} + [C]\{\dot{T}\} = \{f_g\}$$

where the characteristic or conduction matrix

$$[K_1] = k\iint_A \left[\frac{\partial[N]}{\partial x}\frac{\partial[N]^T}{\partial x} + \frac{\partial[N]}{\partial y}\frac{\partial[N]^T}{\partial y}\right] dA \quad (7)$$

$$[K_2] = k\int_{x=0}^{a}\left[[N]\frac{\partial[N]^T}{\partial x}\right]_{y=b} dx \quad (8)$$

$$[K_3] = k\int_{x=0}^{a}\left[[N]\frac{\partial[N]^T}{\partial x}\right]_{y=0} dx \quad (9)$$

The convection matrix

$$[K_4] = h\iint_A [[N][N]^T]_{x=a} dA \quad (10)$$

The capacitance matrix

$$[C] = \rho c\iint_A [N][N]^T dA \quad (11)$$

The gradient matrix

$$\{f_g\} = hT_\infty\int_{y=0}^{b} [[N]]_{x=a} dy \quad (12)$$

B. Element Formation

Following Robert Cook et al [8] and Singiresu Rao [10], for the formulation of the element shape function, express for the field variable in the polynomial form.

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$$\phi(x, y) = a_0 + a_1 x + a_2 y$$

which satisfied the coordinate of the vertices of the triangle are (x_1, y_1), (x_2, y_2), (x_3, y_3) and the nodal conditions

$$\phi(x_1, y_1) = \phi_1, \quad \phi(x_2, y_2) = \phi_2, \quad \phi(x_3, y_3) = \phi_3.$$

Apply Least –Square method and find the value of the shape function for the triangular element (three nodes) is expressed as

$$\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = [1 \quad x \quad y] \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}^{-1}$$

On solving one obtain the interpolation function for three nodes triangular element as

$$N_1(x, y) = \frac{1}{2A}[(x_2 y_3 - x_3 y_2) + (y_2 - y_3)x + (x_3 - x_2)y]$$

$$N_2(x, y) = \frac{1}{2A}[(x_3 y_1 - x_1 y_3) + (y_3 - y_1)x + (x_1 - x_3)y]$$

$$N_3(x, y) = \frac{1}{2A}[(x_1 y_2 - x_2 y_1) + (y_1 - y_2)x + (x_2 - x_1)y]$$

where the area of the triangular element is given as

$$A = \frac{1}{2}\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$$

The structure of the three nodes triangular element is shown in Figure 2(a) and (b), the vertices of the triangle is a, b, c and the arrow indicates the direction of the edges.

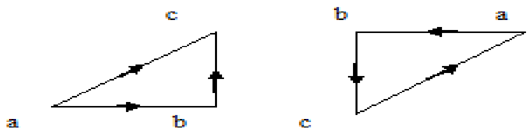

Fig. 2: (a) "Structure of an odd element"
(b) "Structure of an even element"

IV. RESULTS AND DISCUSSION

The objective of this experiment is to obtain the numerical solutions like temperature distribution, displacement at the nodes and thermal stress analysis of a rectangular plate with finite thickness. In this experiment *5456* aluminum alloy plate were used as the work piece. Considering the aluminum rectangular plate of length (a) = *2* m, breath (b) = *1* m and thickness (c) = *0.1* m. The experimental conditions and thermal properties of the aluminum plate is shown in the table 1, which are used for the numerical calculation. Detailed composition and weight percentage of the *5456* aluminum plate are shown in Table *2*.

Table1. Thermal Properties of the 5456 aluminum plate

Thermal conductivity	k (W/m-°C)	116
Density	ρ (kg/m^3)	2700
Specific heat	c (j/kg-°C)	1066
Young's modulus of elasticity	E (GPa)	69
Linear coefficient of thermal expansion	α (1/°C)	23.1×10^{-6}
Poisson's ratio	υ	0.33
Heat transfer coefficient	h (W/m^2 –°C)	10

Table 2. Composition and weight percentage of 5456 aluminum plate

Composition	Al	Mn	Mg	Cr
Weight percentage	93.9	0.8	5.1	0.12

A. *Temperature Distribution*

a. *Discretization of an element*

The rectangular plate is discretized into 45 numbers of nodes (M) and 64 numbers of triangular elements (NE) shown in Figure 3.

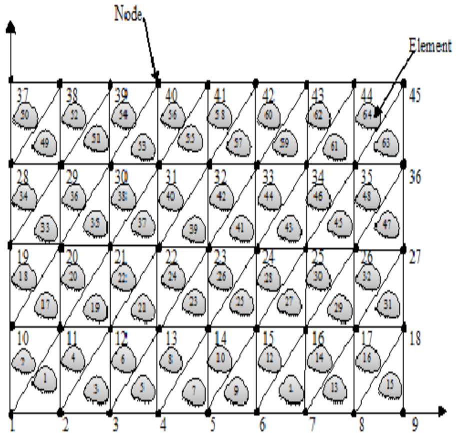

Fig. 3: "Geometry showing node and element position in 2D rectangular plate"

Solving the equations (7), (8), (9), (10), (11), (12), get the elemental conductance matrix, convective matrix, capacitance matrix and force or load vector matrix for all elements

b. *Assembly procedure*

All the 64 elements are assembled using Matlab programming and formed the final equation as given below

$$[K]\{T\} + [C]\{\dot{T}\} = \{f_g\}$$

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Applying the Finite difference as in David Hutton [4] and substituting $\{\dot{T}\} = \left\{\frac{T(t+\Delta t)-T(t)}{\Delta t}\right\}$, the above equation reduce to

$$[K]\{T\} + [C]\left\{\frac{T(t+\Delta t) - T(t)}{\Delta t}\right\} = \{f_g\}$$

This simplified as under

$$T(t+\Delta t) = [C]^{-1}\{f_g\}\Delta t - [C]^{-1}[K]\{T\}\Delta t + \{T\}$$

Solving the above equations by using Matlab programming, one obtains the temperatures at all nodes at different time.
The constant temperature $T_0 = 180\ C^o$,
Initial temperature $T_i = 185\ C^o$ and
Surrounding temperature $T_a = 100\ C^0$.
The numerical value of temperature distribution at time $(t = 10\ sec)$ in the rectangular plate is given in the table 3 and graphical representation is shown in the fig. *4*. The variation of temperature is observed with respect to time. The temperature is high near insulated surface and low near convective surface, it is decreases right to left.

Table 3. Temperature distribution at time $t = 10\ sec$

X/Y	0	0.25	0.5	0.75	1
0	180	185.037	184.8595	185.5783	180
0.25	180	185.268	184.9171	185.2237	180
0.5	180	185.2939	184.8565	185.2816	180
0.75	180	185.2805	184.8651	185.2849	180
1	180	185.2806	184.859	185.2783	180
1.25	180	185.2936	184.8824	185.3053	180
1.5	180	185.2113	184.8099	185.1966	180
1.75	180	185.4736	185.2798	185.3474	180
2	180	184.4005	183.6727	183.715	180

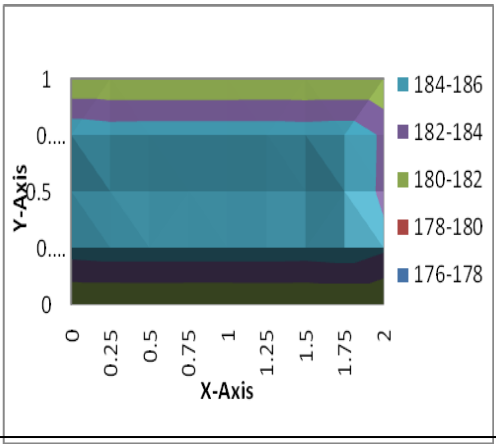

Fig. 4: Temperature distribution in the rectangular plate at time $t = 10\ sec$

B. Displacement analysis

A. Discretization for displacement element

The displacement components of node j are taken as q_{2j-1} in the x direction and q_{2j} in the y direction. It denotes the global displacement vector as $Q = [q_1, q_2, q_3, \dots \dots \dots q_{2M}]^T$ is shown in the Fig. *5*.

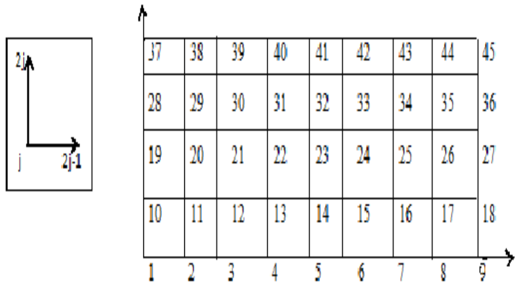

Fig. 5: "Geometry showing nodal displacement"

B. Formation of elemental matrix

Following Robert cook et al. [1], to find the displacement, the element stiffness matrix is given by

$$[K^e] = c_t A_e B^T D B$$

As the distribution of the change in Temperature $\Delta T(x,y) = \Delta T = T_{ij} = T_i - T_j$ is known, the strain due to this change in temperature can be treated as an initial strain ϵ_0 and the element temperature load matrix θ^e can be represented as

$$[\epsilon_0] = [\alpha\Delta t, \alpha\Delta t, 0]^T \text{ and } [\theta^e] = c_t A_e B^T D \epsilon_0$$

where $[D] = \frac{E}{(1-v^2)}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix}$

$$[B] = \frac{1}{\det J}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} \text{ and}$$

$$[J] = \begin{bmatrix} x_{13} & y_{13} \\ x_{23} & y_{23} \end{bmatrix}$$

where the notation, $x_{ij} \triangleq x_i - x_j$ and $y_{ij} \triangleq y_i - y_j$.

C. Assembly Procedure

Assembling all the elemental stiffness and temperature load matrix, the final stiffness matrix K and temperature load matrix θ are formed and expressed as

$$KQ = \theta$$

Applying Gaussian elimination method to solve the above equation and yield the displacement Q at the nodes along X and Y directions are given in the tables 4(a) and

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4(b) and graphical presentation as shown in the Fig. *6(a)* and Fig. 6*(b)*, respectively. From Fig. 6*(a)* it is observed that the displacement high at the centre, near insulated surface and upper right part and low in the right lower and upper region of the rectangular plate along *X* direction. Fig. 6*(b)* shows the displacement along *Y* direction is high at the centre, top and bottom region and low in right middle part of the rectangular plate.

Table 4(a). Displacement along X direction at time $t = 10\ sec$

X/Y	0	0.25	0.5	0.75	1
0	-4.6E-05	-2.3E-05	-1.1E-05	2.34E-05	2.29E-05
0.25	-8.4E-05	6.87E-05	3.05E-05	-3.2E-05	-9.9E-05
0.5	2.29E-05	-5.3E-05	6.1E-05	-3.4E-05	-6.1E-05
0.75	-3.1E-05	-7.6E-06	1.14E-05	-2.5E-05	-4.2E-05
1	-1.5E-05	-7.6E-05	3.81E-06	-1.5E-05	-2.7E-05
1.25	2.29E-05	-2.3E-05	7.63E-06	-1.4E-05	-3.8E-05
1.5	1.53E-05	-2.3E-05	7.63E-06	-5.9E-05	3.81E-06
1.75	7.63E-06	0	-8.4E-05	-4.1E-05	8.01E-05
2	3.05E-05	-7.6E-06	-3.1E-05	-9.1E-06	7.63E-05

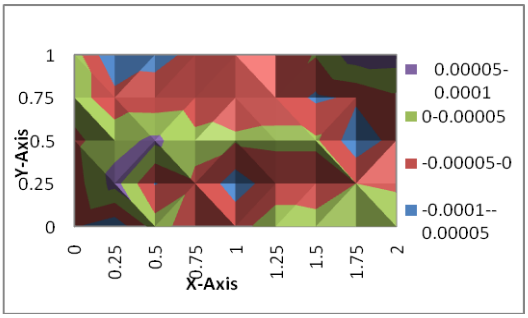

Fig. 6(a): Displacement along X direction

Table 4(b). Displacement along Y direction at time $t = 10\ sec$

X/Y	0	0.25	0.5	0.75	1
0	9.16E-05	-1.5E-05	-0.0005	1.53E-05	3.05E-05
0.25	7.63E-06	-6.9E-05	-0.00018	-8.4E-05	-0.00012
0.5	-8.4E-05	-0.00011	-1.5E-05	-0.00012	-0.00018
0.75	-1.9E-05	-0.00011	-8E-05	-3.1E-05	1.91E-05
1	-9.1E-05	-4.9E-05	-5.6E-06	-2.5E-05	1.91E-06
1.25	-5.7E-05	-4.6E-05	-2.3E-05	-1.9E-05	-2.3E-05
1.5	-6.1E-05	-4.6E-05	0	-7.6E-06	7.63E-06
1.75	-6.1E-05	3.05E-05	6.1E-05	0	-9.2E-05
2	-0.00021	-7.6E-05	0.000214	-1.5E-05	-0.00026

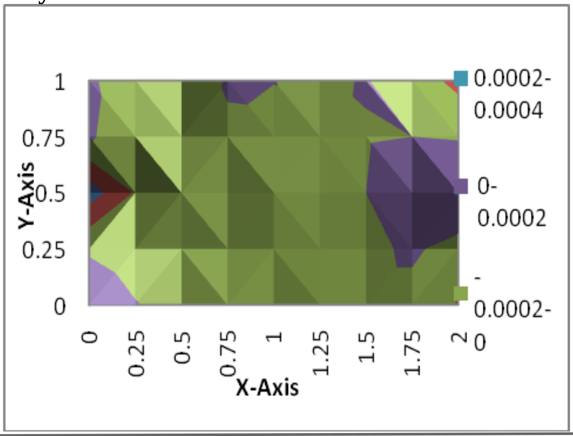

Fig. 6(b): Displacement along Y direction

D. Stress Analysis

Let *u* and *v* be the displacements along *X* and *Y* direction, the displacements inside the element are written using nodal values of the unknown displacement field as

$$u = N_1 q_1 + N_2 q_3 + N_3 q_5$$

$$v = N_1 q_2 + N_2 q_4 + N_3 q_6$$

The shape function represented in area co-ordinates are

$$N_1 = \frac{A_1}{A},\ N_2 = \frac{A_2}{A} \text{ and } N_3 = \frac{A_3}{A}$$

Using the Strain-displacement relation [1] one gets

$$\epsilon = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \end{Bmatrix}$$

$$= \frac{1}{detJ} \begin{Bmatrix} y_{23} q_1 + y_{31} q_3 + y_{12} q_5 \\ x_{32} q_2 + y_{13} q_4 + y_{21} q_6 \\ x_{32} q_2 + y_{13} q_4 + y_{21} q_6 + y_{23} q_1 + y_{31} q_3 + y_{12} q_5 \end{Bmatrix}$$

This equation can be written in matrix form as

$$\epsilon^e = B^e q^e$$

Stress-strain relation is

$$\sigma^e = E(\epsilon^e - \epsilon_0) \text{ Or } \sigma^e = E(Bq - \epsilon_0)$$

where $\epsilon_0 = [\alpha \Delta t, \alpha \Delta t, 0]^T$,

$$\epsilon = [\epsilon_x, \epsilon_y, \epsilon_{xy}]^T \text{ and}$$

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$\sigma = [\sigma_x, \sigma_y, \sigma_{xy}]^T$.

The results of displacements and temperatures at the different nodes are used to find the element thermal stresses along the *X*-axis, *Y*-axis and resultant direction at time $(t = 10\ sec)$ which are shown in the Fig. *7(a), 7(b)* and *7(c)*, respectively. The thermal stress along the *X* direction (σ_x) is high at the middle right part, near the insulated surface and central region shown in Fig. *7(a).* The thermal stress along *Y*-axis (σ_y) is very high in middle right part, high near insulated surface and central part of the plate, low near convection surface shown in Fig. *7(b).* The thermal stress along resultant *X-Y* direction (σ_{xy}) is high in middle right part the plate near the insulated surface and low near convection surface shown in Fig. *7(c).*

Table 5(a). Thermal Stress along X direction at time $t = 10\ sec$

X/Y	0-0.25	0.25-0.5	0.5-0.75	0.75-1
0-0.25	-0.0353	0.01232	0.06376	-0.0511
0.25-0.5	0.0106	-0.0231	-0.0158	0.00997
0.5-0.75	-0.0115	0.00752	-0.0048	0.0019
0.75-1	-0.0102	-0.008	0.00119	0.00816
1-1.25	0.01716	0.01142	-0.0001	-3E-06
1.25-1.5	-0.0017	0.00792	-0.0055	-0.0039
1.5-1.75	0.01795	-0.0267	-0.0231	0.02862
1.75-2	-0.0057	0.05688	0.02127	-0.0491

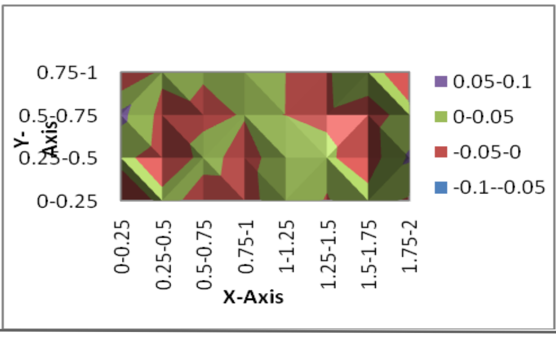

Fig. 7(a): Thermal Stress along x direction

Table 5(b). Thermal Stress along Y direction at time $t = 10\ sec$

X/Y	0-0.25	0.25-0.5	0.5-0.75	0.75-1
0-0.25	0.008569	-0.10896	0.060775	0.009886
0.25-0.5	-0.03574	0.001087	0.020023	-0.02298
0.5-0.75	-0.01521	0.018404	-0.01538	0.001492
0.75-1	-0.01085	0.007451	0.005616	0.013005
1-1.25	0.012819	0.013993	-0.00231	0.002507
1.25-1.5	0.005671	0.006218	-0.00447	0.005706
1.5-1.75	0.007188	0.028223	-0.00622	-0.02499
1.75-2	0.056014	0.018548	-0.06331	-0.0141

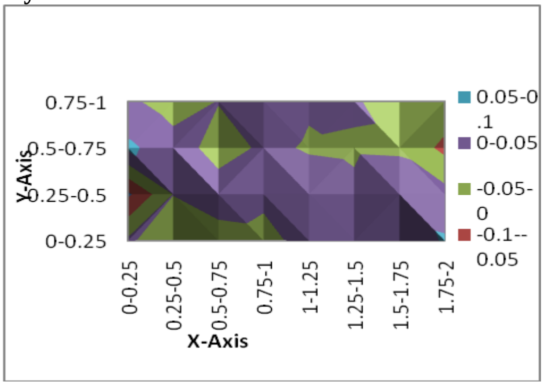

Fig. 7(b): Thermal Stress along y direction

Table 5(c). Thermal Stress along resultant direction at time $t = 10\ sec$

X/Y	0-0.25	0.25-0.5	0.5-0.75	0.75-1
0-0.25	0.001979	0.012468	0.010044	-0.01657
0.25-0.5	-0.00317	0.010291	-0.00145	-0.01003
0.5-0.75	0.000792	0.00376	-0.00545	0.012975
0.75-1	-0.00253	0.012214	0.00128	-0.00212
1-1.25	-0.00361	0.005003	-0.00267	-0.00283
1.25-1.5	-0.00455	0.004354	-0.00282	0.004205
1.5-1.75	0.001583	0.004354	0.002301	0.004824
1.75-2	-0.01583	-0.00317	0.01044	0.001237

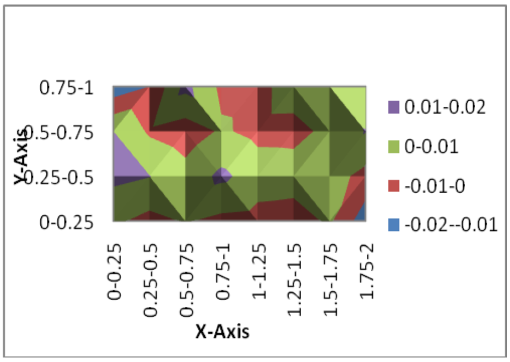

Fig. 7(c): Thermal Stress along resultant direction

VI. CONCLUSION REMARKS

Numerical and experimental studies of the thermal stresses due to temperature changes in the rectangular plate due to convection using finite element method were carried out. The Matlab programming is used for the determination of numerical values for temperature and thermal stresses. Major conclusions of this study are summarizes as follows:

a. A computer program simulates of the two dimensional thin rectangular plate with transient heat conduction is developed. For the better accuracy of the numerical results the large numbers of elements are taken for discretization.

b. The variation of temperature can be observed on the middle of the rectangular plate. The temperature is increased with respect to time high at the central region of the plate and low at the constant temperature surface of the rectangular plate.
c. The maximum displacement is observed at the high temperature region. Also displacement near insulated surface is high and low near convective surface.
d. The development of thermal stresses seen at initial and extreme edges in *X* and Y direction. Also the development of thermal stresses can be observed high near the insulated surface and central region in the rectangular plate.

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